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Effective action of magnetic monopole in three-dimensional electrodynamics with massless matter and gauge theories of superconductivity

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Abstract

We compute one-loop effective action of magnetic monopole in three-dimensional electrodynamics of massless bosons and fermions and find that it contains an infrared logarithm. So, when the number of massless matter species is sufficiently large, monopoles are suppressed and in the weak coupling limit charged particles are unconfined. This result provides some support to gauge theories of high-temperature superconductors. It also provides a mechanism by which interlayer tunneling of excitations with one unit of the ordinary electric charge can be suppressed while that of a doubly charged object is allowed.

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Gauge theories of high-temperature superconductors [1] assume that in certain planar electronic systems spin and charge are separated and the resulting new quasiparticles interact via abelian gauge forces. This idea is seemingly in contradiction with three-dimensional confinement due to magnetic monopoles [2]. One may try to resolve the contradiction by assuming that some of the charged excitations are gapless. Then it is possible that quantum effects of these excitations are sufficiently strong in the infrared to modify the interaction between monopoles in such a way that confinement of charges is lost. To see how this can happen, consider the relativistic version of the problem – three-dimensional quantum electrodynamics of massless bosons or fermions. A simple calculation shows that one-loop contribution of massless charged particles to the gauge field propagator causes its small momentum behavior to change from the usual $1/p^2$ to $1/|p|$. The bilinear part of the corresponding effective action for the gauge field calculated on the monopole configuration then produces a logarithm of the total size of the system. Indeed, the field strength of monopole is $F_{ij} \sim \epsilon_{ijk} x_k / r^3$, therefore,

$$\int d^3x F_{ij} \frac{1}{\sqrt{\partial^2}} F_{ij} \sim \ln \frac{R}{a} \ , \tag{1}$$

where R and a are infrared and ultraviolet cutoffs, respectively. Essentially the same calculation appears in ref.[3]. The coefficient of the logarithm increases proportionally to the number N of massless species. This suggests that in the presence of massless charged particles, at least when N is large enough, monopoles are suppressed via a version of the "infrared catastrophe" and charged particles are unconfined.

The reason why eq.(1) is not sufficient to determine the fate of magnetic monopoles in the presence of massless charged particles even in the large N limit, is that the interaction of a charged particle with monopole has no small parameter, so the bilinear part (1) of the effective action is in no way distinguished relative to terms containing more powers of the gauge field. To make a reliable conclusion, we need all these terms. On the other hand, because the gauge propagator in the large N limit is of order $1/N$, the large N limit suppresses higher-loop contributions to the effective action. Therefore, in this limit the problem reduces to calculation of the one-loop determinants of massless bosons and fermions in the monopole background. For this calculation, we chose to proceed with relativistic three-dimensional particles. Precise dispersion laws for quasiparticles that may occur in real electronic systems are unknown at present. The result and the main steps of our calculation are presented

below. The result confirms the presence of an infrared logarithm in monopole's effective action both in bosonic and fermionic cases. We thus show that single monopoles are suppressed by an "infrared catastrophe" in the presence of a sufficient number of massless matter fields. We cannot state at present what exactly this "sufficient" number is because the answer to this question lies outside the region of validity of the large N approximation.

Consider now the weak coupling limit of three-dimensional electrodynamics when the dimensionful gauge coupling e^2 is much smaller than the ultraviolet scale $M \sim a^{-1}$ at which internal structure of monopole becomes essential. Then, in the absence of massless matter, monopoles and anti-monopoles would form a dilute gas. When the number of massless matter species is sufficiently large, the infrared logarithm causes monopoles and anti-monopoles to assemble into "molecules" – pairs of typical size d that is much smaller than the average distance between the pairs. These pairs interact by a short range potential of order $(d^2/r^2)\ln(d/r)$, so it is natural to expect that charged particles are unconfined. This picture provides some support to the idea of new gauge interactions in planar electronic systems. At the end of this paper we discuss some further applications of our results.

The one-loop contributions to monopole's effective action from a single charged bosonic field and a single charged fermionic field are respectively of the form

$$S_B^{(1)} = \mathrm{Tr}\ln(-D^2) - \mathrm{Tr}\ln(-\partial^2)\ , \qquad S_F^{(1)} = -\mathrm{Tr}\ln(\sigma D) + \mathrm{Tr}\ln(\sigma\partial)\ , \tag{2}$$

where D are covariant derivatives and σ are Pauli matrices. Eq.(2) requires both ultraviolet and infrared regularizations. We compute not expressions (2) directly but rather

$$S_B^{(1)}(R) = \mathrm{Tr}\ln\mathcal{M}_B - \mathrm{Tr}\ln\mathcal{M}_{B0}\ , \qquad S_F^{(1)}(R) = -\frac{1}{2}\left(\mathrm{Tr}\ln(-\mathcal{M}_F^2) - \mathrm{Tr}\ln(-\mathcal{M}_{F0}^2)\right)\ , \tag{3}$$

where

$$\mathcal{M}_B = -\frac{1}{4R^4}(r^2+R^2)D^2(r^2+R^2)\ , \qquad -\mathcal{M}_F^2 = -\frac{1}{4R^4}(r^2+R^2)(\sigma D)^2(r^2+R^2)\ , \tag{4}$$

and $\mathcal{M}_{B0}$ and $-\mathcal{M}_{F0}^2$ are obtained analogously from the operators $-\partial^2$ and $-(\sigma\partial)^2$ of the vacuum sector. This replacement is similar to the one used by 't Hooft in his four-dimensional instanton calculation [4]. If the effective action were not infrared sensitive, the additional factors of $(r^2 + R^2)/R^2$ would cancel between vacuum and

non-vacuum contributions in (3). In the present case, we intend to show that the effective action *is* infrared sensitive. In this case eq.(3) provides an infrared regularization of eq.(2), R being the regulator radius. In what follows we measure all distances in units of R, hence, we set $R = 1$.

The eigenvalue equations for operators (4) are

$$\left(D^2 + \frac{4\lambda}{(1+r^2)^2} \right) \Psi_B = 0 \, , \tag{5}$$

$$\left((\sigma D)^2 + \frac{4\lambda}{(1+r^2)^2} \right) \Psi_F = 0 \, , \tag{6}$$

where λ stand generically for the eigenvalues. In bosonic eq.(5), radial and angular variables are separated by $\Psi_B = \psi(r) Y_{q,l,m}(\theta, \phi)$, where $Y_{q,l,m}$ are the monopole harmonics of ref.[5]. One gets the radial equation

$$\left[\left(\frac{\partial}{\partial r} \right)^2 + \frac{2}{r}\frac{\partial}{\partial r} - \frac{\alpha(\alpha+1)}{r^2} + \frac{4\lambda}{(1+r^2)^2} \right] \psi = 0 \, , \tag{7}$$

where $\alpha = [(l+1/2)^2 - q^2]^{1/2} - 1/2, \;\; l = |q|, |q|+1, ...$, and the multiplicity of the eigenvalue λ is $(2l+1)$. Parameter q assumes integer and half-integer values as a consequence of the Dirac quantization condition. All our results depend only on $|q|$, so in what follows we take $q \geq 0$. In fermionic eq.(6), the variables are separated by using either of the three angular dependences $\xi^{(1)}_{jm}$, $\xi^{(2)}_{jm}$, η_m introduced in ref.[6]. We find that in all three cases the resulting radial equations have the same form as eq.(7) but with different values of α. For angular dependence $\xi^{(1)}$, $\alpha = \mu - 1$, where $\mu = [(j+1/2)^2 - q^2]^{1/2}$ and $j = q+1/2, q+3/2, ...$, while for angular dependences $\xi^{(2)}$ and η, $\alpha = \mu$ with $j = q+1/2, q+3/2, ...$ for $\xi^{(2)}$ and $j = q - 1/2$ for η. In all cases the multiplicity is $(2j+1)$. In three dimensions, fermionic wave functions (or, more precisely, wave sections [5, 6]) Ψ_F in (6) are doublets. This doublet structure is carried by the angular dependences, so both for bosons and fermions radial functions ψ in eq.(7) are one-component objects. Therefore, unlike the scattering problem in (3+1) dimensions [6], our calculation does not require any special treatment of $j = q - 1/2$ fermionic modes.

We can treat bosonic and fermionic cases simultaneously using eq.(7) if we adopt the following notation

$$\alpha = [(j+1/2)^2 - q^2]^{1/2} - \kappa - 1/2 \, , \qquad j = q+\kappa, q+\kappa+1, ... \, . \tag{8}$$

Then, $\kappa = 0$ corresponds to bosons, $\kappa = 1/2$ to fermions with angular dependence $\xi^{(1)}$, and $\kappa = -1/2$ combines fermions with angular dependencies $\xi^{(2)}$ and η. Results for the vacuum sector are obtained by substituting $q = 0$. Though such substitution leads to an unphysical value $j = -1/2$ for $\kappa = -1/2$, eigenvalues corresponding to this unphysical value do not participate in the fermionic trace in (3) because of the vanishing multiplicity factor $(2j+1)$.

By the change of variables

$$\psi(r) = r^{\alpha}(1+r^2)^{-\alpha-1/2}\phi(x) , \qquad x = (1+r^2)^{-1} , \tag{9}$$

eq.(7) is converted into a hypergeometric equation. The resulting eigenvalues are

$$\lambda_n = (n+\alpha+1/2)(n+\alpha+3/2) , \quad n = 0, 1, \dots . \tag{10}$$

We still need an ultraviolet regularization for the traces in eq.(3). A convenient one is provided, again in parallel with 't Hooft's calculation [4], by two Pauli-Villars regulators with masses M_i and metrics e_i satisfying $\sum_i e_i = -1, \ \ \sum_i e_i M_i^2 = 0$, $i = 1, 2$. Regularized traces that we will need are

$$\mathrm{Tr}\ln\mathcal{M}(M_i;\kappa) = \sum_{j=q+\kappa}^{\infty}(2j+1)\sum_{s=1}^{\infty}\sum_{i=0,1,2} e_i \ln[(s+\alpha)^2+\mu_i^2] , \tag{11}$$

where we defined $e_0 = 1$, $M_0 = 0$, and $\mu_i^2 = M_i^2 - 1/4, \ i = 0, 1, 2$. The effective action in the fermionic case is obtained from the half-sum of traces (11) with $\kappa = \pm 1/2$ which are in fact related to each other in a simple way.

Because the effective action of monopole is dimensionless, it can depend only on products of infrared and ultraviolet regulator parameters, $M_i R$. In the system of units where $R = 1$, we are then interested in the dependence of the effective action on M_i in the limit when M_i are large. Non-regulator terms in eq.(11) cannot produce such dependence. Applying the Euler-Maclaurin formula to regulator terms in eq.(11) we get

$$\sum_{s=1}^{\Lambda}\ln[(s+\alpha)^2+M^2] = \int_0^{\Lambda} ds \ln[(s+\alpha)^2+M^2]$$
$$+ \left(\frac{1}{2}\ln[(s+\alpha)^2+M^2] + \frac{1}{6}\frac{s+\alpha}{(s+\alpha)^2+M^2}\right)\Bigg|_0^{\Lambda} + O([\alpha^2+M^2]^{-3/2}) , \tag{12}$$

where $\Lambda \gg M$. The remainder in eq.(12) gives a convergent contribution of order $1/M$ when summed over j with the multiplicity factor $(2j+1)$ and hence may be neglected.

Terms divergent at $\Lambda \to \infty$ as well as those proportional to M_i^2 get cancelled when all regulator and non-regulator contributions are added together and we obtain

$$\mathrm{Tr}\ln\mathcal{M}(M_i;\kappa) = \sum_{j=q+\kappa}^{\infty}(2j+1)\times$$

$$\left[\sum_{i=1,2} e_i\left(-(\alpha+1/2)\ln(\alpha^2+\mu_i^2)+2\mu_i\left(\frac{\pi}{2}-\arctan\frac{\alpha}{\mu_i}\right)-\frac{1}{6}\frac{\alpha}{\alpha^2+\mu_i^2}\right)+C(j)\right] , \tag{13}$$

where $C(j)$ denotes terms independent of M_i.

The dependence of eq.(13) on M_i can be found by the following method. We decompose each sum over j into two – one running from $q+\kappa$ to some value $J-1$ such that $q \ll J \ll M_i$, another running from J to a cutoff $\Lambda' \gg M$. As in the sum over s before, dependence on the cutoff will disappear when all regulator and non-regulator terms are added together. The number J is integer or half-integer when $q+\kappa$ is integer or half-integer, respectively. Now, in the region $q+\kappa \le j \le J-1$ we can neglect j compared to M_i while in the region $J \le j \le \Lambda'$ we can use the Euler-Maclaurin formula. At $J \le j \le \Lambda'$ we can also use the expansion

$$\alpha = \alpha_1 - q^2(2j+1)^{-1} + O(j^{-3}) , \qquad \alpha_1 = j-\kappa . \tag{14}$$

For example,

$$\sum_{j=q+\kappa}^{\Lambda'}(2j+1)(\alpha+1/2)\ln(\alpha^2+M^2)$$

$$= \sum_{j=q+\kappa}^{J-1}(2j+1)(\alpha+1/2)\ln M^2 + \sum_{j=J}^{\Lambda'}(2j+1)(\alpha_1+1/2)\ln(\alpha_1^2+M^2)$$

$$- q^2\sum_{j=J}^{\Lambda'}\left(\ln(\alpha_1^2+M^2)+\frac{\alpha_1(2\alpha_1+1)}{\alpha_1^2+M^2}\right) + O(M^{-2}) + O(J^{-1}) . \tag{15}$$

It turns out that other terms in eq.(13) do not produce contributions in either boson or fermion determinant that distinguish between monopole and vacuum sectors. Proceeding with eq.(15), we finally obtain the one-loop effective action of a monopole with monopole number q in the presence of massless bosons and fermions up to terms independent of R:

$$S^{(1)}_{B,F}(R) = K_{B,F}(q)\ln M^2R^2 + O(R^0) ,$$

$$K_B(q) = \lim_{J\to\infty}\left[\sum_{j=q}^{J-1}(2j+1)[(j+1/2)^2-q^2]^{1/2} - \frac{2}{3}J^3 + \frac{1}{6}J + q^2 J\right] , \tag{16}$$

q	$K_B(q)$	$K_F(q)$
1/2	0.0968	0.0151
1	0.2266	0.1730
3/2	0.3850	0.4358
2	0.5682	0.7852

Table 1: Coefficients of $\ln R^2$ in the one-loop effective action of a monopole with monopole number q in the presence of massless bosons and fermions.

$$K_F(q) = -\lim_{J\to\infty}\left[\sum_{j=q+1/2}^{J-1}(2j+1)[(j+1/2)^2 - q^2]^{1/2} - \frac{2}{3}J^3 + \frac{1}{6}J + q^2 J\right] - \frac{q}{2}\,. \quad (17)$$

We assumed that both regulator masses are of the same order, $M_i \sim M$. The limits in eqs.(16)-(17) were done by computer. The results for a few values of q are presented in Table 1.

When there are N species of particles of given type, the corresponding numbers from Table 1 should be multiplied by N. For sufficiently large N, logarithms of R coming from boson or fermion determinants will overpower $3\ln MR$ that comes with the opposite sign from the volume factor. Thus, when there is a large number of massless matter species, monopoles are suppressed. For small N, we cannot draw any conclusions from the present work because the one-loop calculation is not a reliable guide in this case. Possibly, numerical simulations can help to solve the problem for small N.

Let us now state some applications of our results. They show that non-confining abelian gauge interactions are possible in (2+1) dimensions when gapless excitations are present, thus providing some support to the idea of new gauge interactions in planar electronic systems. In a somewhat different interpretation, our results provide a mechanism by which interlayer tunneling of excitations with one unit of the ordinary electric charge is suppressed while that of a doubly charged object is allowed. Recent work [7] shows that a viable theory of high-temperature superconductors can be constructed if such mechanism is assumed to exist. Let us postulate that a planar system describing one layer in a layered material supports quasiparticles that carry magnetic flux with respect to the new gauge field. There are two varieties of such quasiparticles corresponding to positive and negative fluxes, respectively. Assume further that these quasiparticles carry ordinary electric charge ("holons") which has

the same sign both for positive and negative fluxes. Events of interlayer tunneling of these quasiparticles are described by monopoles and anti-monopoles in three dimensions. If there are also excitations of another sort (”spinons”) that carry charge, rather than flux, with respect to the new gauge field and are gapless, tunneling of a *single* flux can be suppressed by their infrared effects as discussed above, while a *pair* of positive and negative fluxes can tunnel freely.

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